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Discharge coefficients of CFVN predicted for high Reynolds numbers based on Low-*Re*-calibration

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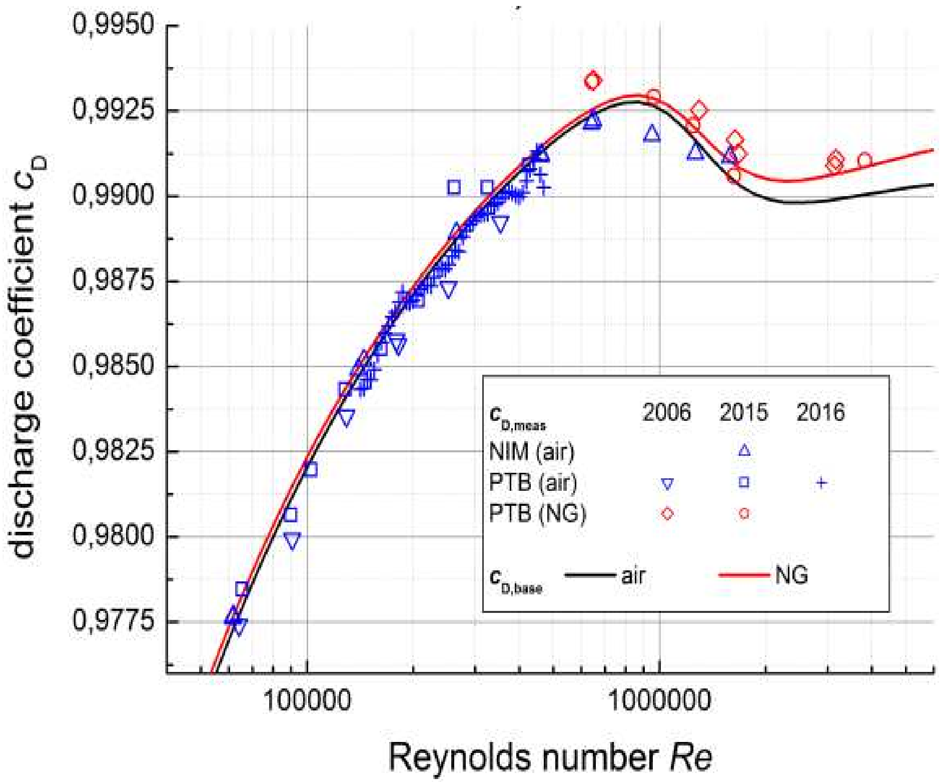
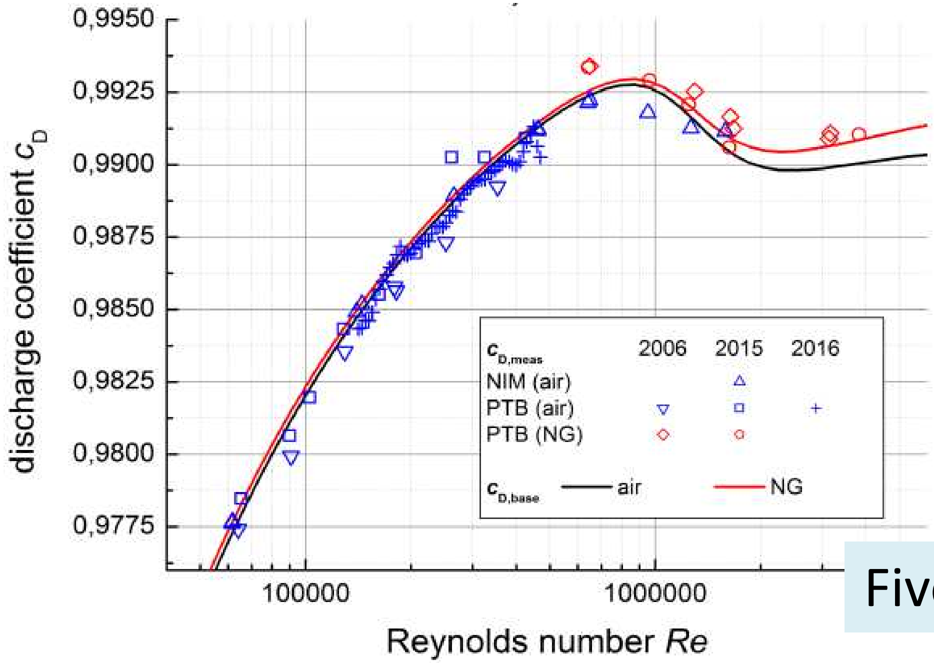
Motivation

Extended data analysis of bilateral comparisons with air and natural gas up to 5 MPa
FLOMEKO 2016



- Successful introduction of a model $c_D = f(Re)$ across laminar and turbulent
- Using only a few parameters
- Can we enhance this approach to extrapolate fom LP to HP?

2016: Description of $c_D = f(Re)$ across transition



$$c_{D,lam} = a - b_{lam} \cdot \mathrm{Re}^{-0.5}$$

$$c_{D,turb} = a - b_{turb} \cdot \mathrm{Re}^{-0.139}$$

$$c_D = s_{lam} \cdot c_{D,lam} + s_{turb} \cdot c_{D,turb}$$

$$s_{lam} = 0.5 \left\{ 1 - \tanh \left[k_u \log \left(\frac{Re}{Re_{tr}} \right) \right] \right\}$$

$$s_{turb} = 1 - s_{lam}$$

Five parameters: a, b_{lam}, b_{turb}, k_u, Re_{tr}

- Fixed relation between b_{lam} and b_{turb} $\quad b_{turb} = 0.003654 \cdot b_{lam}^{1.736}$
- Assuming k_u as a fixed global, common parameter for all nozzles.

$\Rightarrow$ **Reduction to 3 parameter/nozzle: a; b_{lam}; Re_{tr}**

Can we reduce further down to only one parameter?

Further Parameter Reduction

$$c_D = a - b_{lam}/Re^{0.5}$$

Kliegel 1969

$$a = 1 - a_2 + a_3 - a_4$$

$$a_2 = \frac{\kappa + 1}{96(2R_{C,throat} + 1)^2}$$

$$a_3 = \frac{(\kappa + 1)(8\kappa - 27)}{2304(2R_{C,throat} + 1)^3}$$

$$a_4 = \frac{(\kappa + 1)(754\kappa^2 - 757\kappa + 3633)}{276480(2R_{C,throat} + 1)^4}$$

Note on Re_{tr}: also fixed on empirical base to Re_{tr} = 1.25·10^6

Geropp 1971+87

$$b_{lam} = G \cdot R_{C,throat}^{0.25}$$

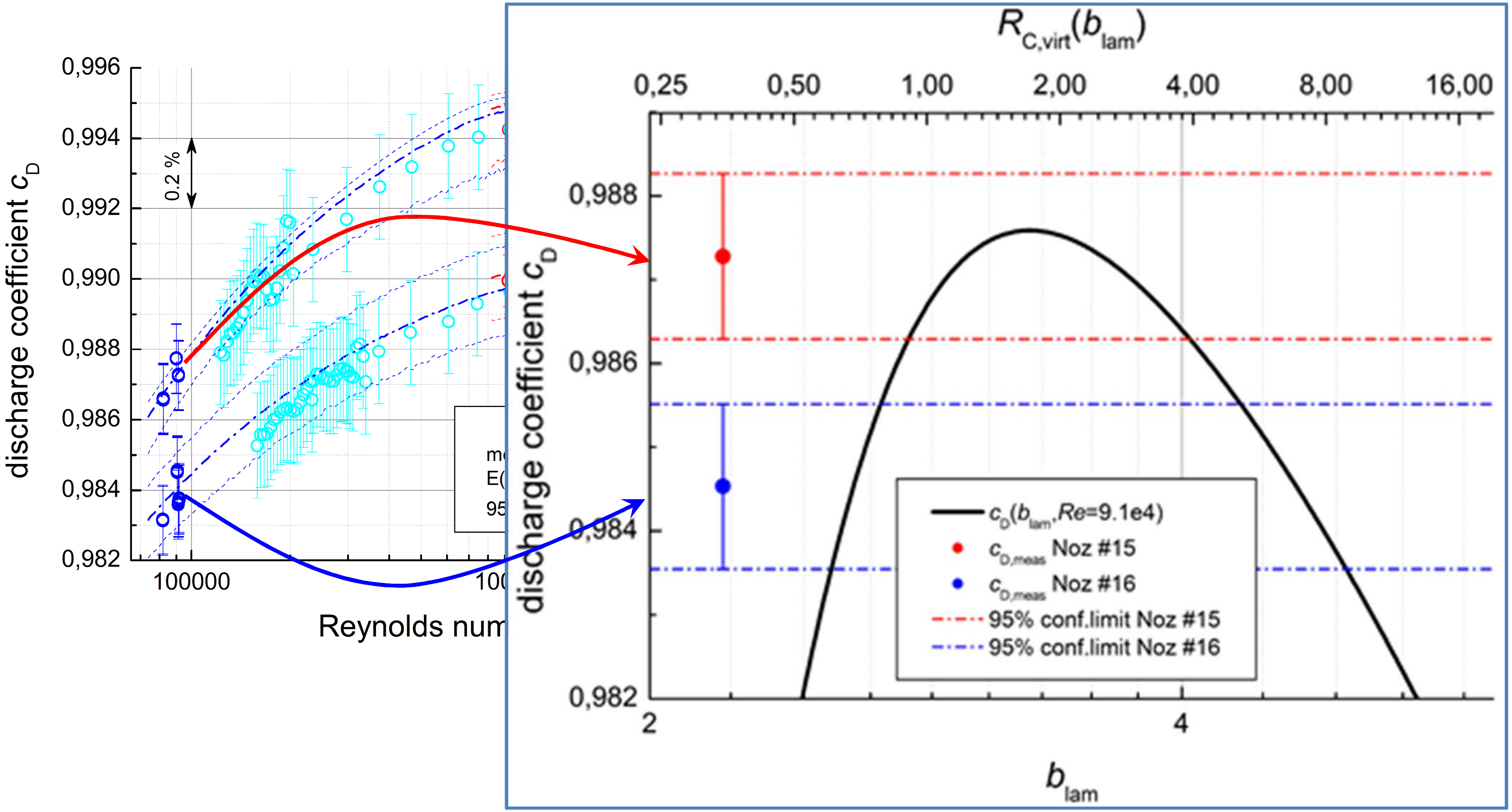
- Common parameter $R_{C,throat}$
- real curvature radius determining these parameters probably differs from $R_{C,throat}$
- We assume that for both the same (virtual) curvature Radius $R_{C,virt}$ applies

$$R_{C,virt} = \left(\frac{b_{lam}}{G}\right)^4$$

$$b_{lam} \xrightarrow[(7)]{} R_{C,virt} \xrightarrow[(5)]{} a$$

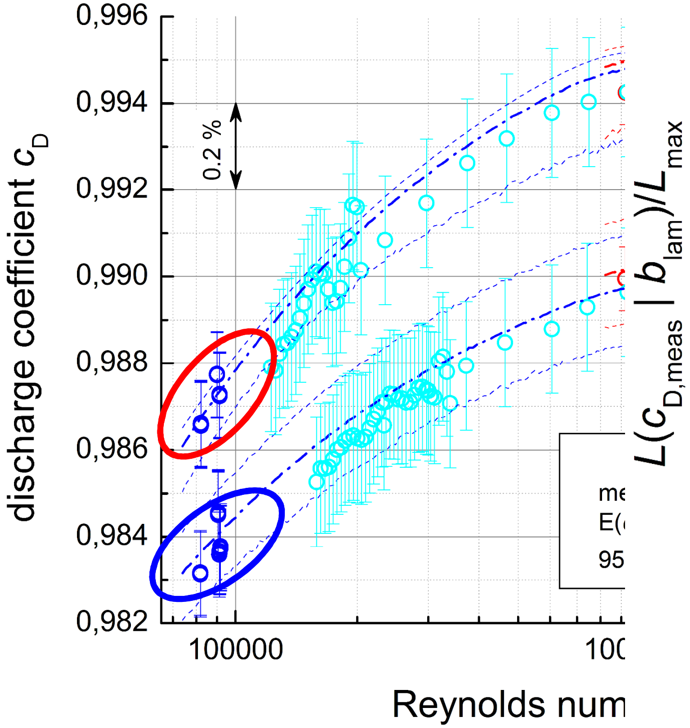
$$b_{turb} = 0.003654 b_{lam}^{1.736}$$

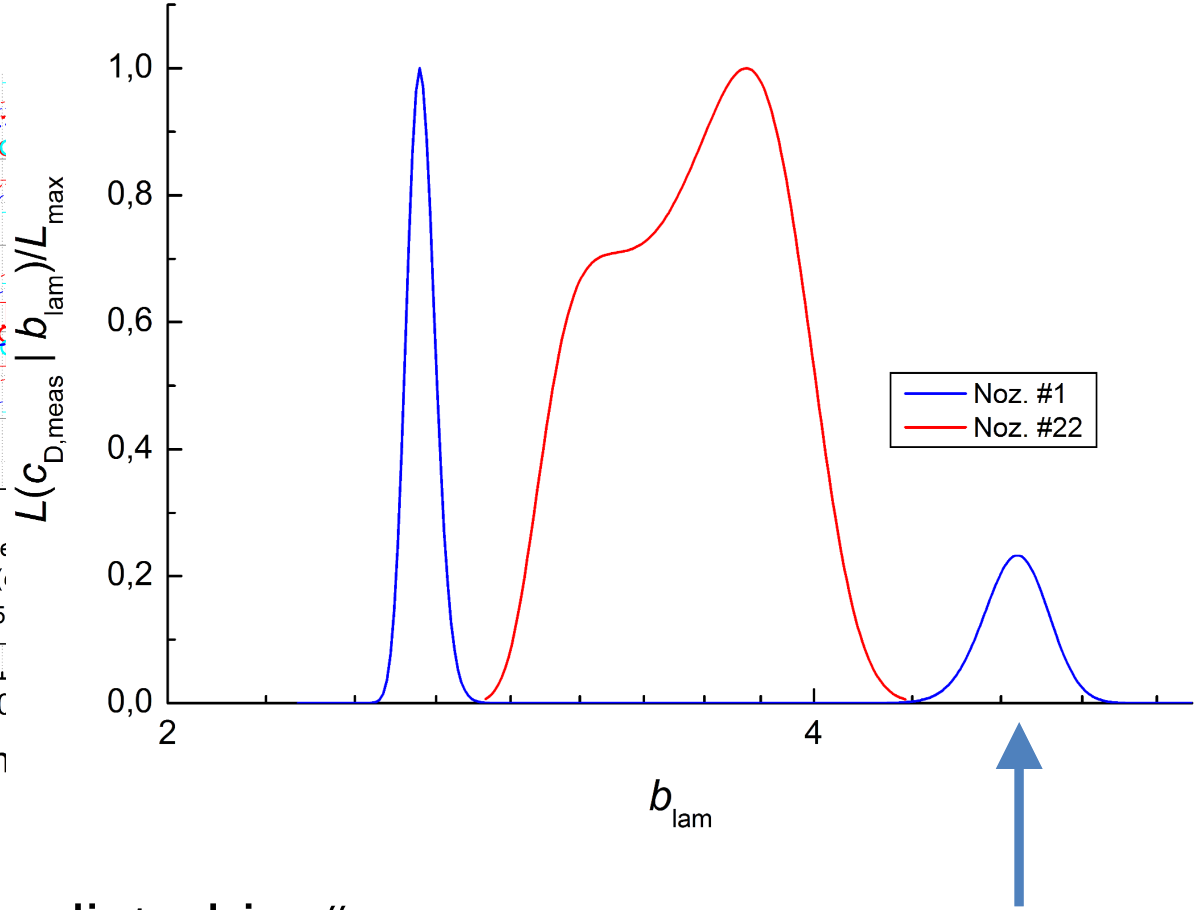
c_D as a function of b_{lam} at a specific *Re*-number



For each measured c_D we can determine the Likelihood $L(c_{D,meas}|b_{lam})$

Likelihood for parameter determination

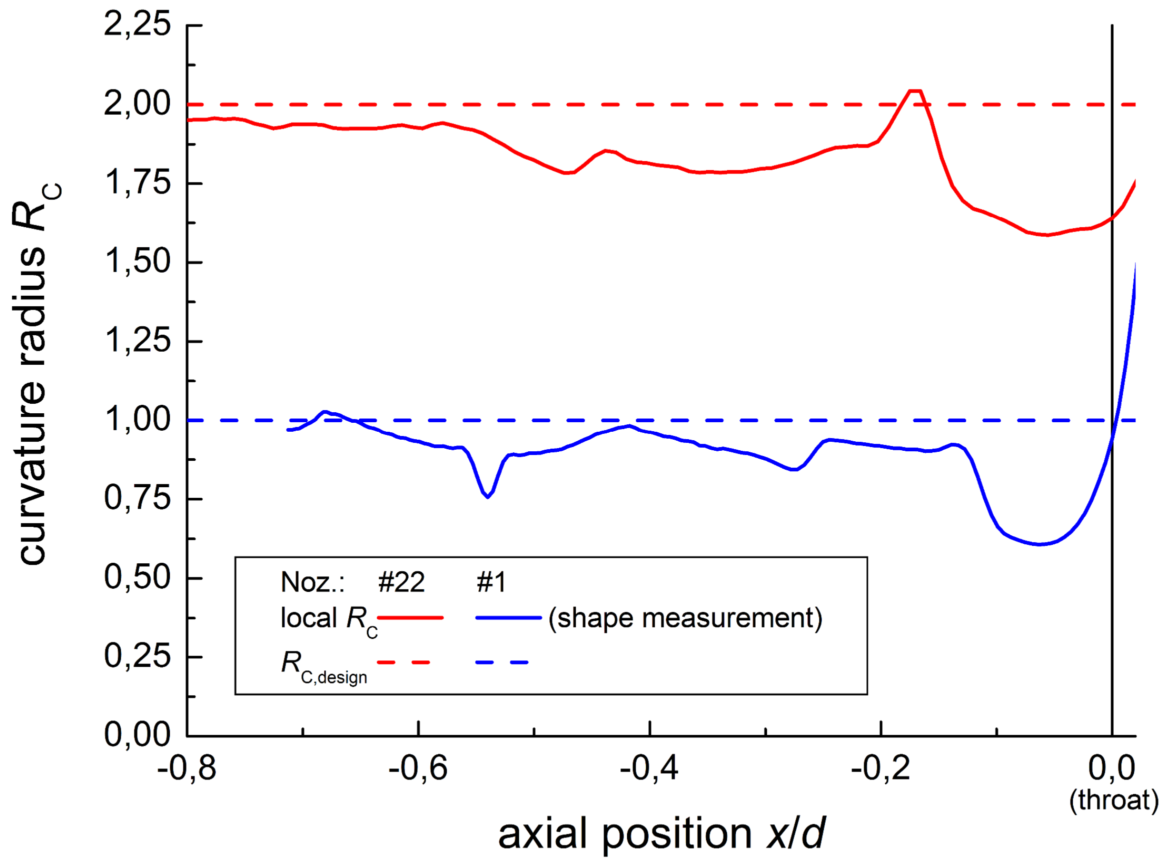


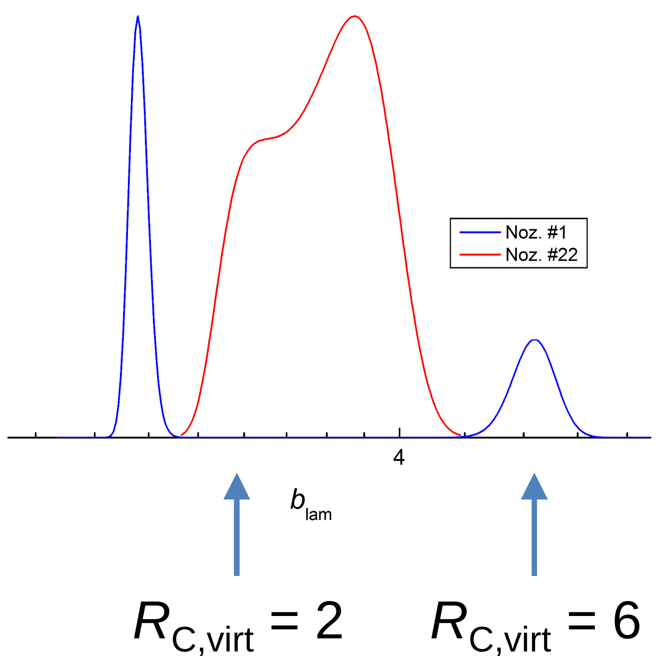


Secondary peak is „disturbing“

=> we needed further information to enhance the outcome

Likelihood for parameter determination

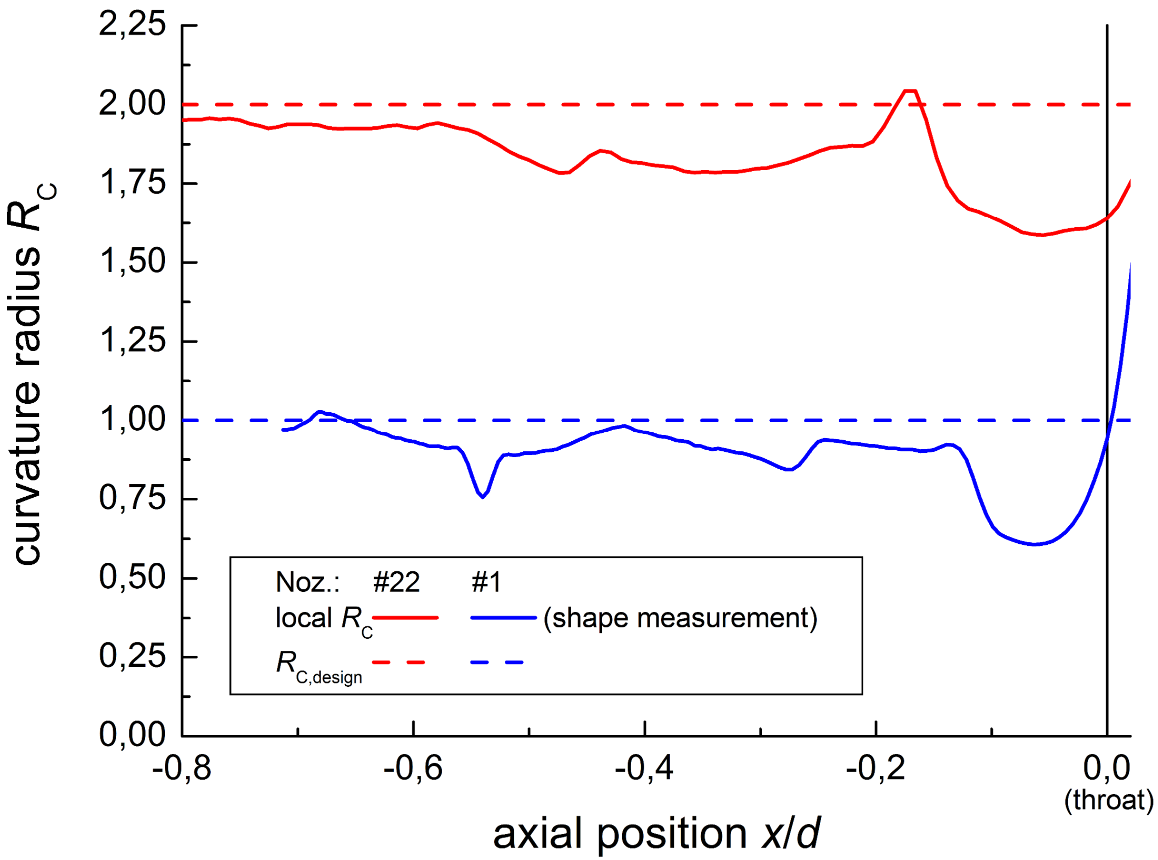


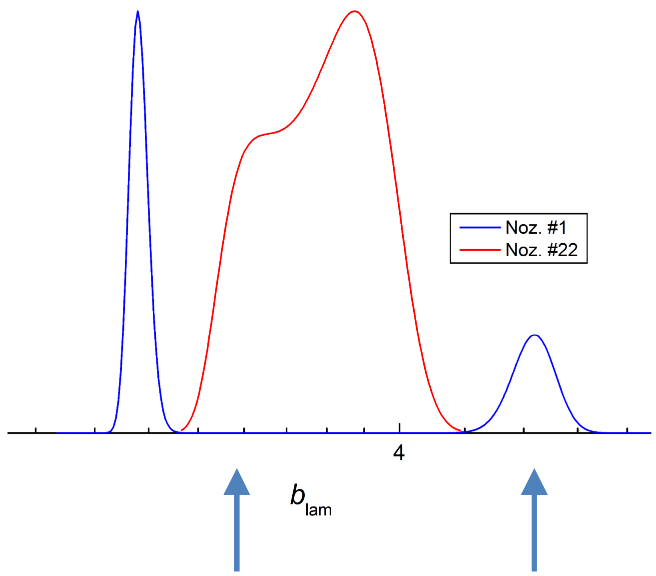


Based on the shape information, it was reasonable to exclude unrealistic parts. The mathematical way to do this is the Bayesian approach:

$$p(b_{\mathrm{lam}}|c_{\mathrm{D,meas}}) \sim L(c_{\mathrm{D,meas}}|b_{\mathrm{lam}}) \cdot p_{\mathrm{prior}}(b_{\mathrm{lam}})$$

Likelihood for parameter determination



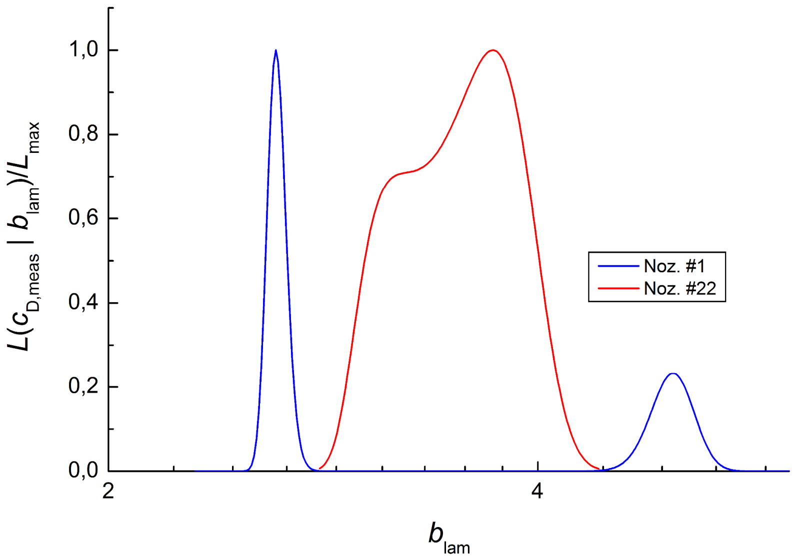


$R_{C,virt} = 2$ $R_{C,virt} = 6$

p_{prior} = const for $0 \leq R_{C,virt} \leq 2 \cdot R_{C,design}$
Zero otherwise

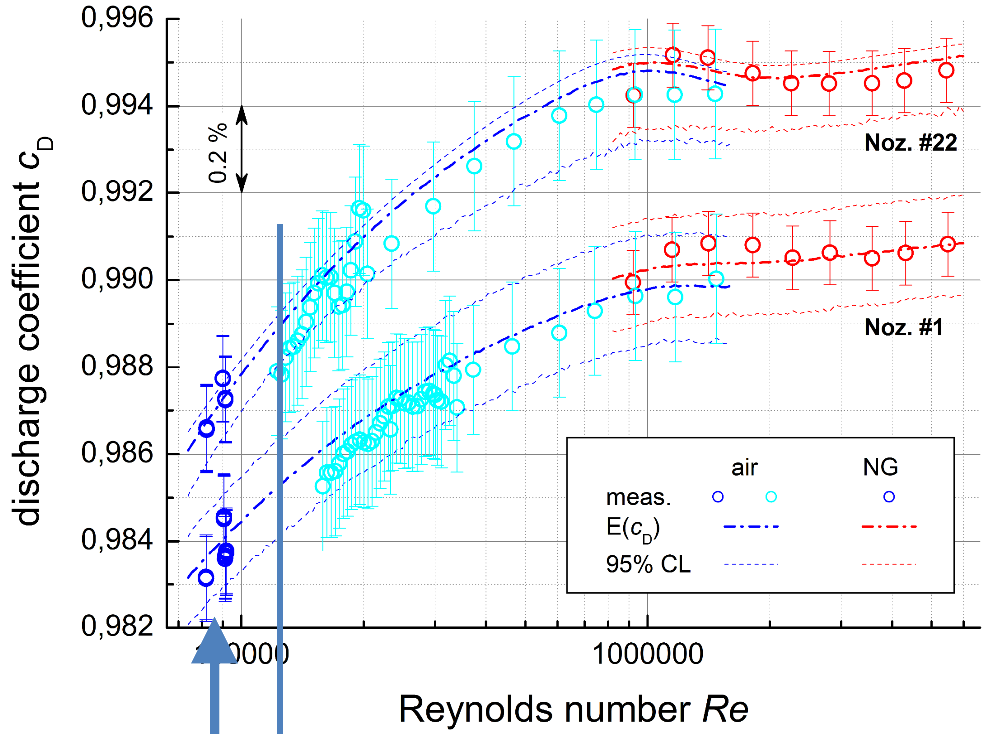
$$p(b_{lam}|c_{D,meas}) \sim L(c_{D,meas}|b_{lam}) \cdot p_{prior}(b_{lam})$$

Likelihood for parameter determination



$$p\left(b_{\mathrm{lam}} \middle| c_{\mathrm{D,meas}}\right)$$

$$E\left(c_{\mathrm{D,pred}}\right)$$



Used for determination

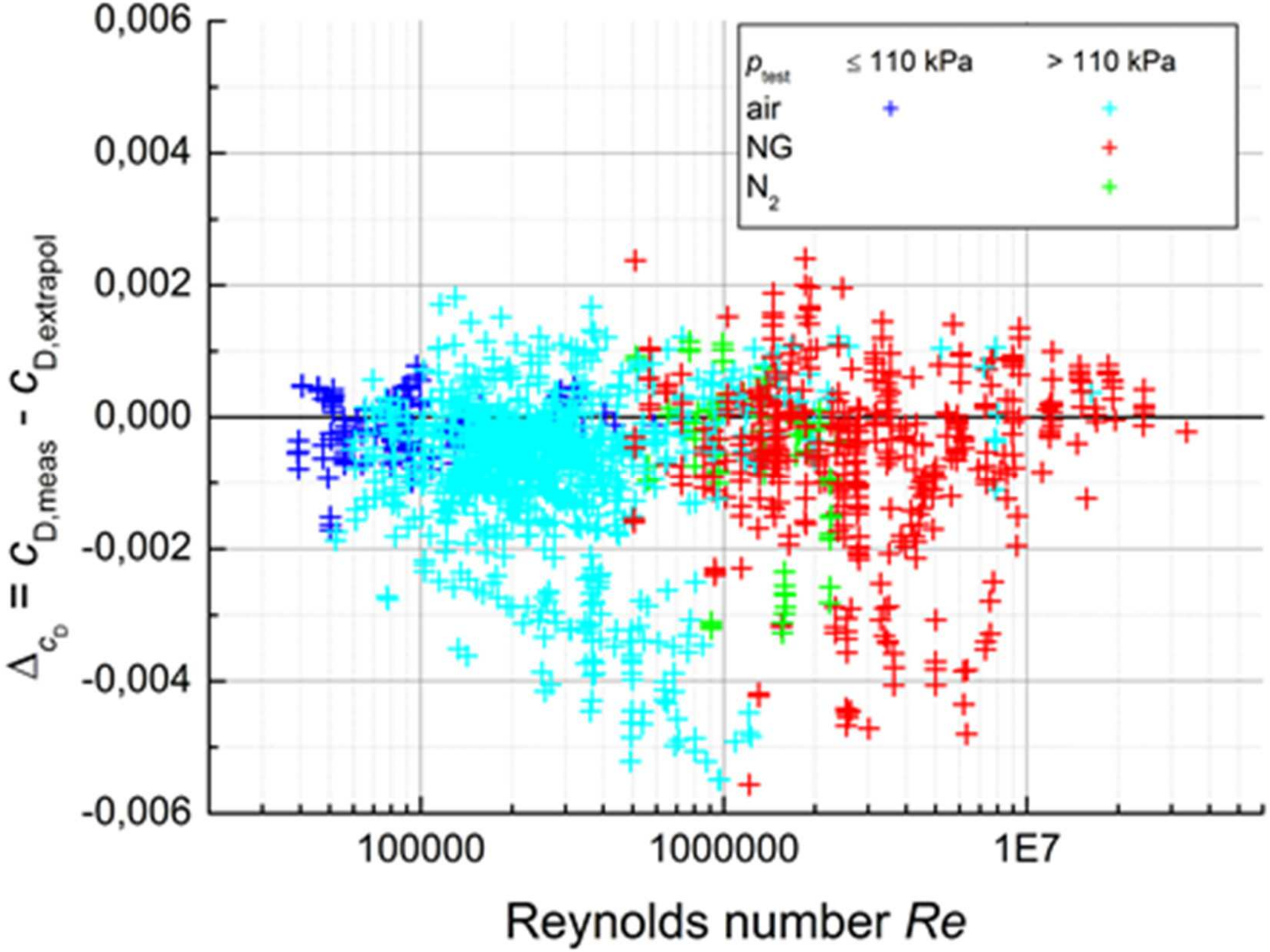
Used for evaluation

Data Base

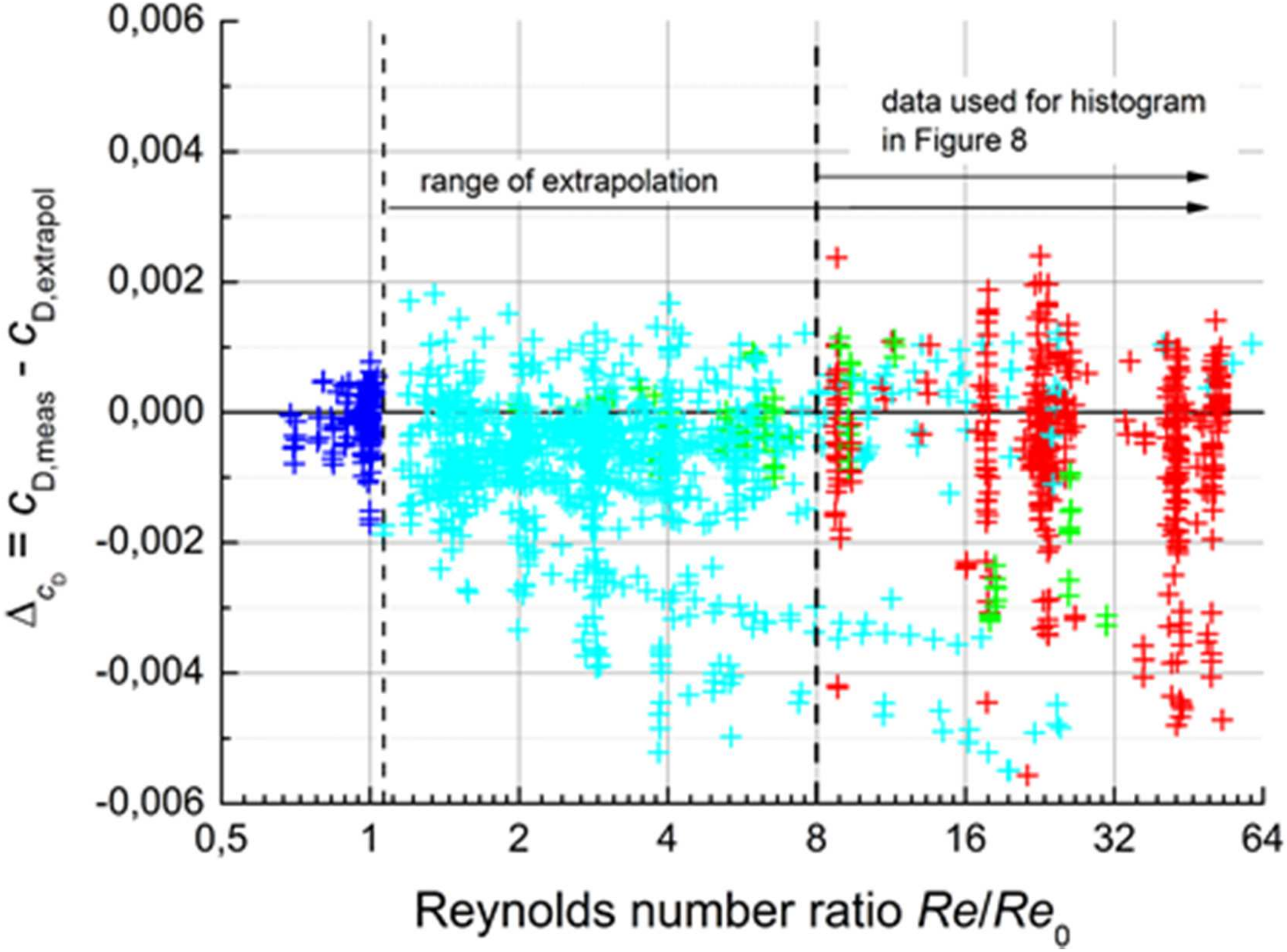
Noz. Nr.	d_{throat} in mm	$R_{C,design}$	Re_0 (100 kPa, air)	for $Re/Re_0 \geq 8$; $k = 2$	
				$U_{Q,min,rel}$	$U_{Q,max,rel}$
1	7,098	1	9,20E4	0,07%	0,15%
2	10,780	1	1,40E5	0,16%	0,16%
3	15,250	1	1,98E5	0,16%	0,16%
4	21,320	1	2,76E5	0,16%	0,16%
5	26,950	1	3,49E5	0,16%	0,16%
6	34,280	1	4,45E5	0,16%	0,16%
7	46,560	1	6,04E5	0,16%	0,16%
8	3,808	2	4,94E4	0,07%	0,15%
9	3,893	2	5,05E4	0,15%	0,15%
10	3,897	2	5,05E4	0,19%	0,24%
11	3,903	2	5,06E4	0,15%	0,15%
12	4,331	2	5,62E4	0,19%	0,20%
13	4,344	2	5,63E4	0,19%	0,22%
14	4,938	2	6,40E4	0,12%	0,12%
15	4,945	2	6,41E4	0,08%	0,14%
16	5,486	2	7,11E4	0,19%	0,25%
17	5,492	2	7,12E4	0,18%	0,23%
18	6,142	2	7,96E4	0,08%	0,12%
19	6,657	2	8,63E4	0,15%	0,15%
20	6,987	2	9,06E4	0,17%	0,19%
21	6,988	2	9,06E4	0,08%	0,14%
22	7,027	2	9,11E4	0,07%	0,15%
23	7,453	2	9,66E4	0,07%	0,15%
24	7,768	2	1,01E5	0,08%	0,12%
25	9,911	2	1,29E5	0,12%	0,12%
26	11,258	2	1,46E5	0,15%	0,15%
27	12,293	2	1,59E5	0,16%	0,18%
28	15,478	2	2,01E5	0,10%	0,12%
29	19,376	2	2,51E5	0,10%	0,12%
30	24,551	2	3,18E5	0,10%	0,12%
31	31,253	2	4,05E5	0,12%	0,12%
32	25,400	2,5	3,29E5	0,18%	0,19%
33	10,000	cylindr.	1,30E5	0,07%	0,13%

- 7 nozzles with $R_{C,design} = 1$
  24 with $R_{C,design} = 2$
  1 with $R_{C,design} = 2.5$
  1 cylindrical nozzle
- Diameter from 3.8 to 46.6 mm

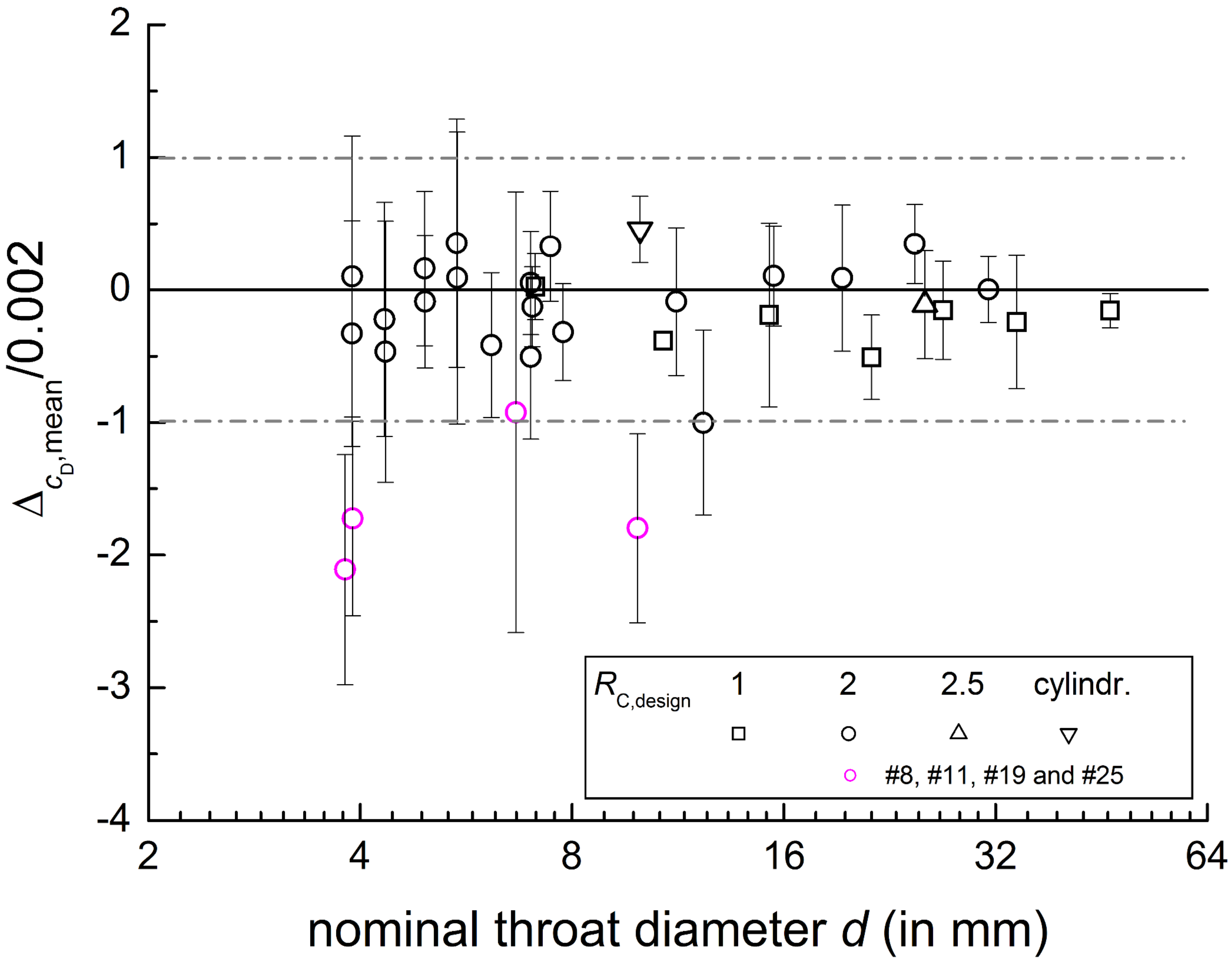
Evaluation: $\Delta c_D = c_{D,meas} - c_{D,extrapol}$



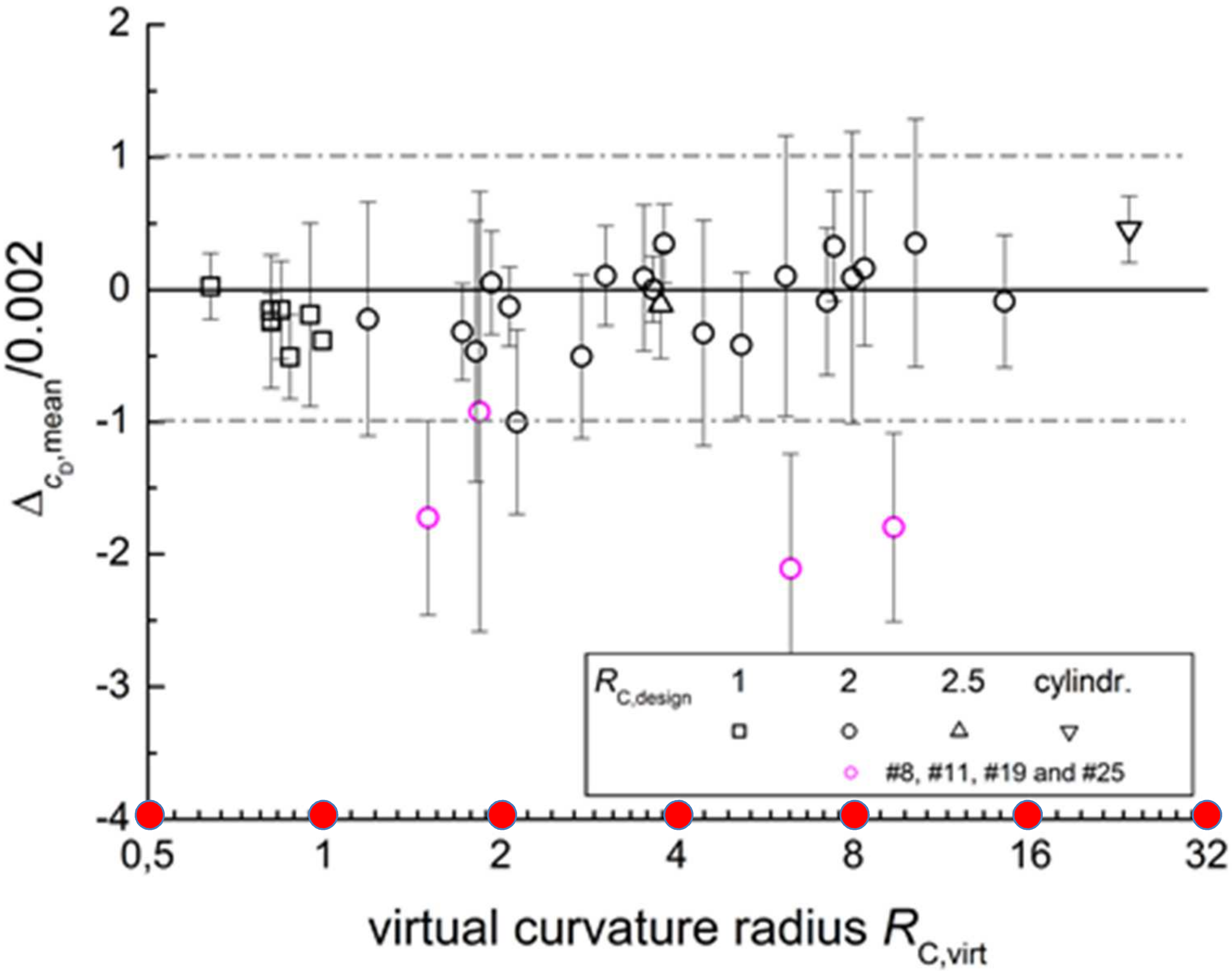
Evaluation: $\Delta c_D = c_{D,meas} - c_{D,extrapol}$



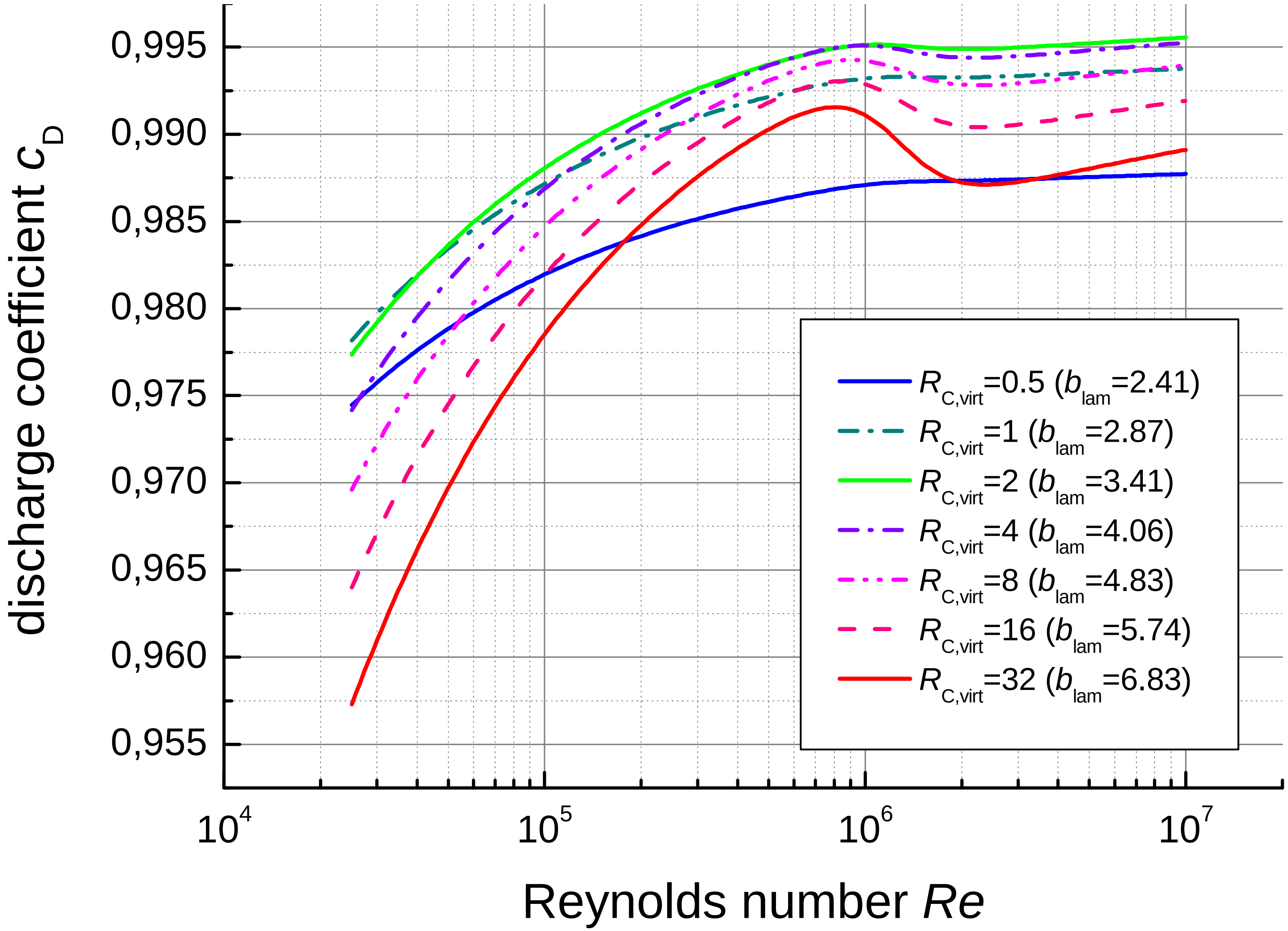
Evaluation: $\Delta c_{D,mean}$



Evaluation: $\Delta c_{D,mean}$



Range of c_D-values covered



Summary and Outlook

- c_D-model with only one free parameter works reliable for 29 of 33 very different nozzles with $R_{C,design}$ from 1 to cylindrical.
- Extrapolation by factor of 60 in Reynolds up to $Re = 3 \cdot 10^7$ is confirmed.
- The $R_{C,virt}$ (and with this the c_D vs. Re) differs sometimes dramatically from the design value.
- Model is restricted to hydraulic smooth surfaces.
- Approach will be extended into two directions:
  - usage of LP(Air)-calibration in wider a *Re*-range (e.g. 1:8) => diameter can also be derived from flow measurements.
  - Bayesian approach will be refined to make better use of prior i.e. information out of dimensional measurement (diameter, shape).
- We can link our HP-traceability with the LP-traceability (see paper 1075, session S16.1, „Combining three independent traceabilities…“)

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